Loughborough University

Analytical Research Project for M.Sc. Business Analytics

Scheduling overnight charges for a fleet of electric vehicles

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Submission Date: 18th September 2024

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1. Abstract

A crucial aspect of current fleet management is the optimization of overnight charging schedules for electric vehicle (EV) fleets. The goal is to minimize operating costs while guaranteeing that cars are prepared for their daily tasks. This work investigates the creation and use of a hybrid strategy that combines heuristic techniques with mixed-integer linear programming (MILP) to produce economical and effective charging solutions. The main goal is to prioritize station-based charging in order to reduce the need for more costly public charging infrastructure. A Greedy Heuristic and Random Search method were created using Xpress Workbench and Python to quickly provide charging schedules that are almost ideal. The outcomes show that the heuristic techniques may guarantee that all vehicles are properly charged while significantly lowering charging expenses.

The study concludes that giving station-based charging priority is a workable and economical approach to managing an EV fleet. The research also sheds light on possible areas for future development to increase scalability and flexibility, such as the incorporation of more sophisticated heuristics and the investigation of dynamic scheduling strategies.

2. Introduction

Reducing carbon emissions and advancing toward sustainable transportation have made the switch to electric vehicles (EVs) a top priority. Electric vehicles (EVs) present a viable substitute for internal combustion engine (ICE) cars in light of the rising concerns over climate change and the depletion of fossil fuels. However, there are a number of practical issues with the widespread use of EVs, particularly for large fleets. One such issue is scheduling their charging. In addition to satisfying operational demands, effectively managing the charging of EV fleets minimizes energy costs, minimizes reliance on costly public charging infrastructure, and eases pressure on the power grid. As the number of EVs in fleets increases, the need for optimal scheduling systems becomes more pronounced (Alonso et al., 2014).xd

When it comes to charging management, fleet operators confront a number of significant obstacles. First and foremost, they have to make sure that every car has enough charge to last through the next day's operations. This is crucial for fleets in sectors like ride-sharing, public transportation, and logistics. Second, operators need to keep the total cost of billing as low as possible. These costs include both the direct costs of electricity and the indirect costs of using public chargers, which are often more expensive than private charging stations (Houbbadi et al., 2019). Furthermore, because the capacity of the infrastructure used for charging is frequently constrained, it is crucial to prioritize which cars should be charged at fleet depots and which, if required, should be charged at public stations.

Scheduling EV charging becomes more complex when factoring in time-of-use (TOU) electricity prices, which fluctuate throughout the day and can significantly impact overall costs if not managed properly (Liu et al., 2021).

Several methods have been suggested by earlier research to improve EV charging schedules. These strategies span a variety of techniques, from more advanced heuristic-based algorithms like Genetic Algorithms (GA) and Greedy Heuristic approaches to more conventional optimization techniques like Mixed-Integer Linear Programming (MILP). MILP is a popular method for solving scheduling problems because it models the problem as a set of linear equations with integer constraints and is good at finding optimal solutions. However, while MILP is powerful, it can be computationally expensive and impractical for large fleet operations (Alonso et al., 2014). Due to this constraint, heuristic methods have been investigated; while they may not always yield the best answer, they do provide near-optimal results in a significantly shorter amount of time. For instance, GA has been shown to be effective in managing the trade-offs between computational time and solution quality by simulating the process of natural selection (Liu et al., 2021).

In situations when choices must be made fast and with little computational power, greedy heuristics in particular are frequently employed. In the context of EV charging, a Greedy Heuristic prioritizes station-based charging over public charging, thereby reducing the overall charging costs while ensuring that all vehicles are adequately charged (Houbbadi et al., 2019). This approach can be both a benefit and a drawback because it bases pricing decisions on the best option at the time rather than always taking the long term into account. This method offers a quick and scalable solution, but it might not always result in the best use of resources, particularly when there is a capacity constraint on charging stations or when the energy needs of the vehicles vary.

The restricted quantity of charging stations at fleet depots has a significant role in the scheduling of EV charging. Due to its affordability and internal management of the charging process, station-based charging is largely relied upon by fleet operators. All cars cannot be charged concurrently at the depot due to restricted station capacity. This constraint forces some vehicles to use public charging stations, which are often subject to availability and higher costs (Houbbadi et al., 2019). Multiple studies have attempted to solve the problem of optimising EV charging for big fleets in recent years. The optimisation of electric bus (EB) fleets was investigated by Houbbadi et al. (2019), with an emphasis on reducing battery degradation costs by strategically arranging charging sessions. Their research showed that fleet operators might save money over the long run by increasing the life of their EV batteries and lowering their immediate operating expenses through smart scheduling of charging. Similar to this, Liu et al. (2021) suggested a bilevel programming approach that responds to TOU energy pricing and allocates EVs to a limited number of chargers in an optimum manner. This model showed how charging cars off-peak and reducing the need for public chargers might result in considerable cost savings.

Apart from these techniques, Random Search has become a useful heuristic tool for large-scale optimisation issues. The method by which Random Search operates is by producing many random solutions and picking the top-performing one. Even while this approach cannot ensure that the best solution will be found, it can produce a workable solution fast, particularly when combined with other optimisation strategies as MILP and GA (Liu et al., 2021). Because Random Search doesn't require the processing overhead of more conventional approaches, it may be used to search for potential answers more broadly in situations when the issue space is too big to be examined completely. Random Search can assist in determining workable charging schedules for EV fleets that minimise expenses while guaranteeing that every car is adequately charged.

This project's main objective is to create an optimisation framework for scheduling EV fleets' nighttime charging. This framework will give a thorough solution to the charge problem by combining the techniques of MILP, Greedy Heuristic, and Random Search. As a baseline, the MILP model will provide the best possible solution to the issue. To deliver near-optimal answers more quickly, the study will also use heuristic techniques in recognition of the scaling constraints of MILP. The goal of these heuristic techniques is to prioritise station-based charging in order to minimise total expenditures and lessen reliance on public chargers. The study will also investigate how GA may be used to simulate natural selection and gradually evolve better schedules, hence improving the quality of the solutions.

With the use of Python and Xpress Workbench, these optimisation techniques will be put into execution to provide fleet operators with an effective way to manage their electric vehicle fleets. The project's main goal is to keep charging expenses to a minimum while making sure that every car has enough power to run. Additionally, the project will examine the trade-offs between computing time and solution quality, offering important new information on the best approaches to large-scale EV fleet management. The project's output is expected to add to the expanding literature of research on EV fleet management by providing useful suggestions for scheduling charging times most effectively in realistic situations (Houbbadi et al., 2019; Liu et al., 2021).

To sum up, effective scheduling of electric vehicle charging is essential to cutting expenses and guaranteeing fleet preparedness. In order to solve the difficulties associated with EV charging scheduling for big fleets, this project will provide an optimisation framework that integrates GA, MILP, Greedy Heuristic, and Random Search techniques. The study's findings will be very helpful in determining the best practices for managing EV fleets, with an emphasis on cutting expenses and decreasing dependency on public charging infrastructure.

3. Literature Review

The transportation and energy sectors are facing new issues as electric vehicles (EVs) become more widely used. With the world moving away from fossil fuels and towards lowering carbon emissions, EVs are now essential to reaching sustainable transportation targets. The increased use of EVs, especially in big fleets, poses serious difficulties for charging schedule optimisation. These difficulties are essential to maintaining grid stability, energy consumption control, and cost effectiveness. This review of the literature critically assesses the research that has been done on the scheduling of EV charging, with an emphasis on optimisation techniques like machine learning, heuristic methods, and meta-heuristic algorithms like Genetic Algorithms (GAs). It also looks at how various approaches handle real-world problems including battery management, time-of-use (TOU) power pricing, and restrictions in the charging infrastructure.

3.1 Heuristic and Meta-Heuristic Approaches for EV Charging Scheduling

Heuristic techniques, including the Greedy Heuristic and meta-heuristic algorithms like GAs, are extensively utilised in the resolution of complex and dynamic EV charging scheduling issues. These techniques are useful for managing non-linear and multi-objective situations, particularly when dealing with big EV fleets and smart grids. The efficiency of GAs in particular has been shown in maximising load patterns and guaranteeing EV integration into current power systems.

An optimisation method utilising GAs was created by Alonso et al. (2014) to control EV charging in residential areas. The study stressed that in order to lessen peak demand and slow down power system ageing, flattening transformer load profiles in low-voltage systems is crucial. The strategy was created to prevent expensive grid infrastructure changes by dynamically adapting charge schedules to variations in grid demand. According to Alonso et al. (2014), the use of GAs made it possible to make real-time modifications to charging schedules, avoiding overloads and preserving grid stability. This study illustrates how heuristic techniques, which provide advantages like peak shaving and lower power losses, can optimise EV charging in limited locations.

Large-scale electric bus (EB) fleet management is another area where heuristic approaches are being used more frequently. Using GAs, Gao, Zhang, and Wang (2019) created an optimisation model for electric bus charging schedules. Their model took power losses, voltage variations, and fluctuations in energy prices into consideration while minimising operating expenses. Through the use of GAs, the researchers were able to show that these algorithms are especially useful in large-scale settings, where traditional optimisation techniques are difficult due to energy consumption fluctuation, like bus depots. This study emphasises the heuristic approaches' scalability in real-world electric vehicle fleet management, particularly in public transportation where operational readiness and cost efficiency are critical (Gao, Zhang, and Wang, 2019).

3.2 Techniques for Optimising Large-Scale EV Fleet Management

The scheduling issue is made more difficult by sizable EV fleets, such as electric bus fleets. Various considerations, including vehicle battery condition, limited charging station capacity, and shifting energy prices, must be considered when managing charging schedules for big fleets. A major obstacle in managing a big fleet of vehicles is making sure that every vehicle is charged within the allotted time frame without going over the charging station's capacity. In order to reduce bus fleet charge costs, Rinaldi, Ferreti, and Lanza (2013) suggested a mixed-integer linear programming (MILP) approach.

According to Rinaldi, Ferreti, and Lanza (2013), the MILP model efficiently managed the scheduling of EVs by assigning charging slots while minimising energy usage and charging infrastructure expenses.

Although MILP models are durable, they can be computationally expensive, especially when used for large fleets. Therefore, as alternatives to get near-optimal solutions while reducing computational complexity, heuristic techniques like Greedy Algorithms have been presented. In order to minimise peak loads and save operating expenses, Jahic, Milinkovic, and Vuckovic (2019) created a greedy algorithm to optimise the charging schedule of a sizable fleet of electric buses. According to Jahic, Milinkovic, and Vuckovic's 2019 paper, heuristic approaches can effectively provide outcomes without the computational strain of MILP, which makes them appropriate for real-time fleet management applications.

Battery ageing is a crucial component of fleet management. Over time, frequent cycles of charging and discharging can deteriorate battery performance and shorten the overall life of electric vehicle batteries. In order to solve this problem, Wang et al. (2020) created a Markov Decision Process (MDP) model that was able to optimise 16,359 electric buses' charging schedules over 1,400 bus routes. Their methodology aimed to minimise battery wear and charging expenses while maintaining operational readiness.

In addition to considering the long-term effects of battery deterioration, the MDP model offered a scalable method for controlling fleet-wide charging (Wang et al., 2020). This study emphasises how crucial it is to include battery management in EV charging plans, especially for large fleets where replacement battery costs can mount up.

3.3 Infrastructure Restrictions' Function in Charging

A common constraint on managing an EV fleet is the availability of charging infrastructure. Even though fleet depots usually have designated charging areas, there might not be enough of them to accommodate large fleets' needs, particularly during peak hours. Although there are versatile, public charging stations which are typically more expensive and might not always be available. For operational efficiency and cost reduction, it is important to assign automobiles between private and public charging stations in an efficient manner.

A scenario-based stochastic model was presented by Yang, Liu, and Zhang (2021) to solve the problem of inadequate charging infrastructure. Their model used k-means clustering to produce realistic scenarios in order to optimise the charging schedule for electric buses with a restricted number of charging stations. By increasing computational efficiency, this method made large-scale applications possible. Fleet managers were able to efficiently deploy charging resources while reducing expenses because to the model's reliable approach for handling schedule unpredictability (Yang, Liu, and Zhang, 2021). This work provides a useful method for resolving charging infrastructure constraints in large-scale fleet operations by including scenario-based modelling.

Similarly, Zhang, Cai, and Song (2019) investigated how to include TOU power price into EV parking lot charging schedules. By planning charging at off-peak times when power costs are lower, their methodology maximised the charging process. To cut down on charging expenses even more, the model also included renewable energy sources like photovoltaic (PV) systems. According to Zhang, Cai, and Song's 2019 study, fleet operators may save a lot of money on operating expenses by integrating renewable energy and TOU pricing into their charging schedules. This approach also helps fleet operators achieve sustainability goals.

3.4 Machine Learning and Predictive Models in EV Charging

There are now more options for optimising EV charging schedules because of recent developments in machine learning (ML). In order to provide more precise scheduling based on battery health, ML models have been used to forecast the Remaining Useful Life (RUL) of EV batteries. Fleet managers will find these predictive models especially helpful as they allow for proactive battery management and lower the likelihood of unplanned battery failures. Additionally, predictive models provide more accurate scheduling, guaranteeing that cars are charged to maximise battery life while satisfying operational requirements.

In order to optimise the charging schedules for a fleet of lightweight electric delivery trucks, Chen et al. (2020) used machine learning algorithms. To develop the best possible charging schedules, the study took into account battery health and the availability of charging stations. Researchers discovered that ML models decreased total operating costs, increased battery life, and enhanced charging efficiency. This study emphasises how machine learning (ML) may improve EV charging schedule accuracy and efficacy, especially in big fleets where battery management is crucial (Chen et al., 2020).

Future research should focus on the integration of ML with optimisation techniques like GA and MILP. Fleet operators may create more effective charging schedules that take into consideration both short-term operating requirements and long-term battery management by integrating predictive models with optimisation approaches.

3.5 Environmental and Economic Impacts

Electric buses, in particular, provide substantial financial and environmental advantages over traditional fuel-powered automobiles A study by Lajunen (2014) found that hybrid and fully electric buses significantly reduce both operational costs and greenhouse gas emissions. The study did point out that the advantages of electric buses are dependent on the availability of renewable energy sources and the effectiveness of the infrastructure for charging them. Because of this, maximising charging schedules ensures that EVs contribute to a sustainable energy future in addition to cutting expenses (Lajunen, 2014).

Ou, Zhang, and Chang (2010) highlighted this point further by examining the fossil energy consumption and greenhouse gas emissions of alternative fuel buses across their whole life cycle. According to their analysis, the least number of emissions was produced by electric buses—but only when the electricity came from low-carbon or renewable energy sources. This emphasises the necessity of comprehensive solutions that incorporate both vehicle scheduling and more extensive changes to energy policies (Ou et al., 2010).

3.6 Challenges and Future Directions

There are still a number of issues with EV charging optimisation, despite recent improvements. A number of variables, including driving style, weather, and road conditions, can affect how much energy an electric car uses. To increase the precision of energy consumption estimates, future research may need to include dynamic variables and real-time traffic data in their models (Pelletier et al., 2019).

Furthermore, the initial expense of purchasing electric buses and setting up the necessary infrastructure for charging them remain major obstacles to their broad adoption. In order to encourage the transition to electric fleets while mitigating the financial risks associated with new technologies, governments and transit agencies will need to create financing models (Ong, Mahlia, and Masjuki, 2012).

4. Problem Description

The Electric Vehicle Charging Scheduling Problem (EV-CSP) in this project addresses the challenge of scheduling the charging of a fleet of electric vehicles (EVs) using limited charging resources. An integrated charging model with multiple charging options will be developed to determine and allocate charging schedules that minimize operational costs, taking into account the capacity of the charging stations and the availability of public charging infrastructure. The objective is to maximize the operational efficiency of the fleet while minimizing the total charging cost, which includes station and public charging costs.

Depending on availability and constraints on budgets, the fleet operator must schedule charging at either private fleet stations or public chargers to guarantee that all vehicles meet their daily operational requirements. The fleet has three main charging options: Public Charging, Station Charging, and No Charge (for vehicles that have enough charge left). Each vehicle's remaining charge, the operational needs for the following day, the capacity of the charging stations, and the pricing differences between station and public charging are all taken into consideration when determining where and when to charge it.

Public chargers must be used by vehicles that need to be charged but are unable to be accommodated at fleet charging stations because of restricted capacity. However, the optimisation challenge is further complicated by the fact that public chargers are usually more expensive and have less availability certainty. The goal is to reduce dependency on public chargers in order to minimise operating costs, while giving priority to station charging because it is typically more economical.

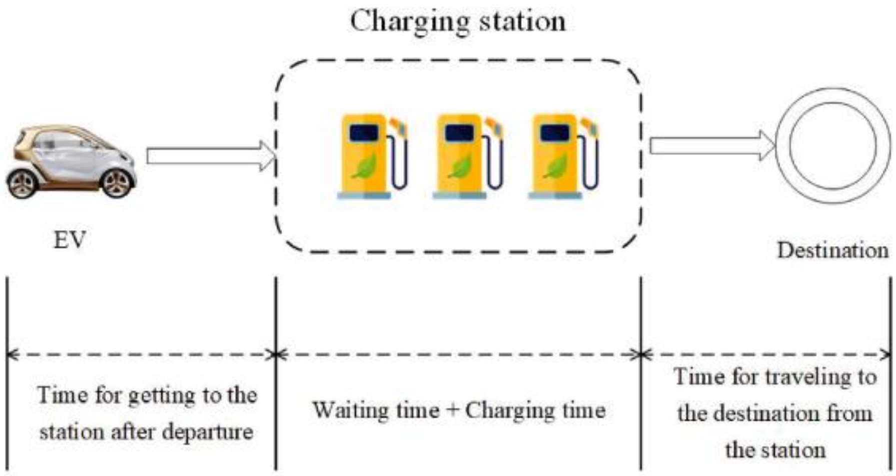

Figure 1: *EVs travel process including charging.*

The **station charging** method is the most economical choice for fleet managers as fleet of vehicles can be charged overnight at designated fleet depots. The main expenses linked to this choice are the Operational Costs, which comprise the expenses related to station maintenance, the electricity consumed during charging, and the Capacity Constraint, which arises from the fact that fleet stations can only accommodate a restricted number of vehicles concurrently. Vehicles that need to be used for early shifts the next day and those with the least amount of charge left get priority.

Public charging second method, which is utilised when the station is completely full. If there are no spaces available at the station, vehicles will have to use public charging stations. If there are no spaces available at the station, vehicles will have to use public charging stations. While this offers flexibility, there are two drawbacks: public chargers may not always be available, adding uncertainty to the charging process, and they cost a lot more than station charging. The fleet manager needs to carefully consider how many cars are using public charging to minimise expenses and maintain operational readiness.

And last, there is **No Charge**, which is a choice for cars that have enough battery life left to finish their trips the following day without requiring more charging. In order to free up station capacity for vehicles that require charging more urgently, these cars won't be charged on a given day. The algorithm will use the remaining energy and the impending operational requirements to determine which vehicles can be left uncharged.

End-Users' Requirements in EV Fleet Charging Optimization

During this study, particular end-user needs for maximising electric vehicle (EV) fleets' overnight charging schedules have been determined. The principal aim was to develop a system that would satisfy the fleet operations' logistical requirements while minimising operational expenses and optimising charging efficiency. Using heuristic algorithms (such as

Greedy) and Mixed-Integer Linear Programming (MILP) models, this study attempted to achieve its objectives by determining both optimal and nearly optimal solutions.

- **Cost Efficiency:** Fleet managers' main goal is to keep charging-related expenses to a minimum. The analysis shows significant cost savings by giving station-based charging priority over public infrastructure. Heuristic algorithms like Greedy as well as the MILP model, both efficiently minimise overall charging expenses, with a focus on reducing the more costly public charging fees.
- **Effective Scheduling:** The capacity to control charging schedules without going over the station's capacity has been identified as one of the essential operating needs. By integrating capacity restrictions over a 21-day period, this study offers models to fleet operators that aid in the optimisation of charging priority and vehicle rotation. Especially the heuristic approaches provide rapid modifications to day-to-day activities, improving decision-making in real time.
- **Sustainability:** Aligning activities with environmental goals is a crucial demand from public sector stakeholders. Station-based charging is supported by the models in this study, and it works well with renewable energy sources. The research is important to public transport authorities that are focused on green transitions since it is in line with policy measures that aim to reduce carbon footprints.

Tools and Software Used

To optimise the charging schedules for electric vehicle (EV) fleets, a variety of software tools and programming environments was used in this study. Each tool was carefully chosen based on its suitability for strategic planning and real-time decision-making, as well as its capacity to handle difficulties related to addressing complicated optimisation problems. An overview of the instruments utilised and the reasoning behind their choice is provided below.

- **Xpress Workbench:** For creating and resolving Mixed-Integer Linear Programming (MILP) models, utilise the Xpress Workbench. Because of its strong performance in managing complex optimisation issues, Xpress was chosen as the best option for locating globally optimal solutions while working within the limitations of the fleet's charging infrastructure.
- **Python:** Because of its speed and versatility, Python was chosen to develop the Greedy Heuristic algorithm. It made use of libraries like NumPy for effective data handling, enabling real-time decision-making and rapid response to change fleet conditions.
- **Microsoft Excel:** Excel was used to prepare and handle the data. Its intuitive interface made it simple to arrange input data, which allowed for a seamless integration into the optimisation models. Examples of this data include vehicle charge levels, station capacities, and cost factors.

5. Methodology

This chapter provides an in-depth explanation of methodology used to solve the Electric Vehicle Charging Scheduling Problem (EV-CSP) for a fleet of electric vehicles (EVs). The project's main objective is to reduce the overall cost of charging while making sure that every vehicle has enough charge to meet its operational requirements by the following day. The strategy used two main optimisation techniques: a Mixed-Integer Linear Programming

(MILP) model created in Xpress and a Greedy Heuristic implemented in Python. Based on computational speed, solution quality, and practicality, a comparative analysis was conducted to evaluate the efficacy of each model that was created to solve the EV-CSP.

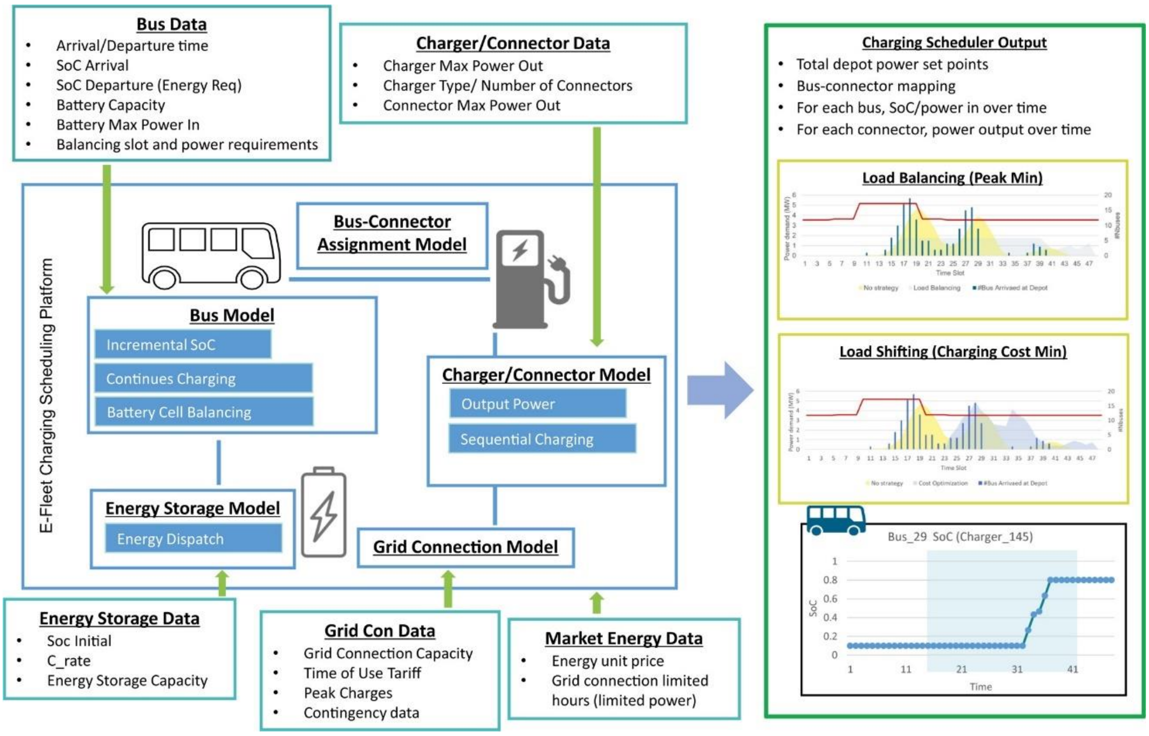

***Figure 2**: Detailed explanation of EV vehicle charging and running cycle*

5.1 Research Approach

Due to the constraints of charging station capacity, time-of-use (TOU) electricity pricing, and the different energy requirements of each car, the problem of optimising EV fleet charging is intrinsically difficult. The requirement for scalable, affordable, and rapid solutions requires the use of two different approaches: a more robust mathematical optimisation strategy combined with a fast heuristic solution. This methodological approach was chosen to make sure the models could handle various real-world situations when decisions needed to be made quickly or completely optimised.

The process included several crucial phases:

1. Data Collection and Preprocessing
2. Implementation of the Greedy Heuristic in Python
3. Development of the MILP Optimization Model in Xpress
4. Comparison and Analysis of Results

5.1.1. Data Collection and Preprocessing

Data on vehicle energy usage, charging station capabilities, and TOU pricing for electricity were needed to solve the EV-CSP efficiently. A fleet of 14 cars was included in the dataset, which was collected over the course of 21 days and included information on things like remaining battery levels, total energy used by each car, and the accessibility of charging stations. The amount of energy required by each vehicle was used to decide whether it required charging on a particular day or not.

The expenses of charging at public and private fleet stations (also known as station charging and public charging, respectively) were also included in the data. The scheduling of charging hours to minimise expenses was influenced by the fluctuating costs of electricity throughout the day, which was reflected in time-of-use electricity pricing. To verify that all of the data points in this dataset were complete and appropriate for analysis, anomalies were removed during the preprocessing stage.

In the given datasets, there are parameters (Base Parameters) of the following.

Base Parameters for Dataset

- **V =** Number of vehicles
  Indicates the total number of electric cars (EVs) in the fleet that needs to have their charging optimised.

- **D** = Number of days
  Represents the number of days in the planning period

- **C** = Charging station capacity
  Represents the maximum number of vehicles that can be charged at the fleet's dedicated charging station at once.

- **fs** = Cost of charging at the station
  The amount paid for every energy unit used at the fleet's charging station during the charging process.

- **fp** = Cost of charging at a public charger
  Represents the amount paid for every energy unit used at the public charging station when there is no space available in charging stations.

- **t** = Running time after full charge
  Represents the number of days vehicle V can run after getting full charged at the station.

- **t0** = Initial charge level
  Represents the number of days vehicle V can still run with it’s initial charging percentage.

Decision variables

- **x(v, d)** = Charging decision at station
  A binary decision variable that takes a value of 1 if vehicle v is charged at the station on day d, and 0 otherwise.

- **y(v, d)** = Charging decision at public charger
  A binary decision variable that takes a value of 1 if vehicle v is charged at a public charging station on day d, and 0 otherwise.

- **total_station_cost** = Total cost incurred by charging vehicles at the station

The total cost, computed over the course of the schedule period, from each vehicle charged at the station.

- **total_public_cost** = Total cost incurred by charging vehicles at public charging stations
  The total cost of using public charges, depending on how frequently and how long they are used.

- **total_cost** = Overall cost of charging
  the total of total_station_cost and total_public_cost, which amounts to the entire cost of car charging during the optimisation period.

Value of t and t0 is shown below. These values were written in excel sheet which was then uploaded in xpress software.

No.	t	t0
1	3	1
2	3	0
3	3	2
4	3	1
5	3	2
6	3	1
7	3	0
8	3	1
9	3	1
10	3	1
11	3	2
12	3	1
13	3	1
14	3	2

Table 1: *Value of t and t0 used in xpress software*

5.2. Model Formulation

A mixed-integer linear programming (MILP) model was developed to address the Electric Vehicle Charging Scheduling Problem (EV-CSP). The model's goal is to reduce an electric vehicle fleet's overall charging costs over a specified time period while guaranteeing that each vehicle is charged effectively to satisfy its operational needs. The MILP model takes into account a number of restrictions, such as the limited capacity of charging stations, the desire to reduce dependency on public charging infrastructure, and time-of-use electricity price.

5.3. MILP Optimization Model in Xpress

A Mixed-Integer Linear Programming (MILP) model was created using Xpress Workbench, an optimisation tool renowned for its capacity to resolve challenging mathematical programming issues, as a supplement to the heuristic approach. By taking into account every possible charge option for the whole fleet during the specified time period, the MILP model is intended to identify the globally optimal solution.

The MILP formulation ensures that the overall cost is minimised while still fulfilling the operating needs of the fleet by taking into consideration the charging limits of the fleet, including station capacity, public charging prices, and TOU pricing. The MILP model, in contrast to the Greedy Heuristic, optimises across all vehicles and days while taking the total solution into account for each decision.

The approach described by Liu and Lamsali (2009), who utilised MILP to location and allocation difficulties, optimising the scheduling of facilities based on capacity and demand, is also in line with the need for optimisation in this project. This fundamental strategy explains why MILP should be used to oversee the restricted number of charging stations while guaranteeing low charging expenses and station overloading.

5.3.1. Rationale for the MILP Model

The MILP method was selected because of its:

Optimality: When long-term cost minimisation is the top goal, it ensures the best feasible solution by applying mathematical techniques to the problem.

Precision: The model ensures that every facet of the charge issue is taken care of because it is quite accurate and accounts for a variety of limits and variables.

Flexibility: In real-world situations where fleet sizes, charging requirements, and costs may differ, Xpress's flexible constraint management and adaptations come in handy.

5.4. Initial Base Model Formulation – Mixed Integer Linear Programming

1. **Step 1: Initialization:** All of the decision variables and parameters are first initialised by the model. This involves establishing the values for V, D, C, fs, fp, and the arrays for battery levels (t and t0).
2. **Step 2: Binary Decision Variables:** Determining whether a vehicle is charged at a specific location on a given day is what the binary variables $x\ (v,d\ )$ and y $(v,d\ )$ represent.
3. **Step 3: Initializing Constraints:** Capacity restrictions and charging specifications are put into practice to guarantee that vehicles are charged correctly and the station's capacity is respected.
4. **Step 4: Optimisation Solver:** To minimise overall billing costs while meeting all limitations, the goal function is passed into the optimisation solver (Xpress or Python-based tools).
5. **Step 5: Output:** The findings include each vehicle's charging schedule, along with information about whether it was charged at a station or a public charger and the overall expenses incurred during the scheduling time.

Step 1 and step 2 are explained in data collection and preprocessing part where all the parameters and decision variables are defined. Next comes step 3 where other parameter and constraints are defined as given below:

5.4.1. Initialization

```
initializations from 'mmodbc.excel:noindex;EV_Charging_Schedule_modified.xlsx'
  t as "[A2:A15]"
  t0 as "[B2:B15]"
end-initializations
```

The goal of this block is to initialise the data and parameters from the EV_Charging_Schedule_modified Excel file.xlsx.

- t: Indicates how long each car can be driven on a full charge. Column A is where the values are read (from cells A2 to A15).
- t0: Taken from column B, this number indicates each vehicle's starting battery charge level (from cells B2 to B15).

When dealing with huge datasets for fleet management, the initialisation step enables the model to read external data straight from a spreadsheet.

5.4.2. Objective Function

```
! Objective function to minimize the total charging cost
Z := sum(v in 1..V, d in 1..D) (fs*x(v,d) + fp*y(v,d))
minimize(Z)
```

Goal: Reduce the overall cost of charging for all vehicles and days.

- fs: The station's charging expense.
- fp: The price of using a public charger for charging (usually more than fs).

The total cost of utilising the station and public chargers is determined by adding together all vehicles (v in 1..V) over all days (d in 1..D). The solver is instructed to minimise the overall cost Z by using the minimize(Z) function.

$$\text{Minimise} \quad \sum_{v=1}^{V}\sum_{d=1}^{D} f_s x_{vd} + \sum_{v=1}^{V}\sum_{d=1}^{D} f_p y_{vd}$$

Subject to

$$\sum_{v=1}^{V} x_{vd} \le C\,, \qquad d = 1, \ldots, D$$

$$1 - \sum_{d=1}^{t_v^0+1} x_{vd} \le y_{v,t_v^0+1}\,, \qquad v = 1, \ldots, V$$

$$1 - \sum_{j=\max\{d-t_v+1,1\}}^{d} x_{vj} \le y_{vd}\,, \qquad v = 1, \ldots, V;\ d = t_v^0, \ldots, D$$

$$x_{vd}, y_{vd} \text{ are binary}, \qquad v = 1, \ldots, V;\ d = 1, \ldots, D$$

The image shared above contain formulation for a Mixed Integer Linear Programming (MILP) issue that aims to minimise the overall cost of charging for a fleet of electric cars (EVs). Let us examine each component of the formulation in more detail:

5.4.3. Constraints

Station Capacity Constraint

$$\sum_{v=1}^{V} x_{vd} \leq C, \quad d = 1, \ldots, D$$

The purpose of this constraint is to make sure that the total number of vehicles charged at the fleet's station on day d never goes above the charging capacity of the station, denoted by C.

- $\sum_{v=1}^{V} x_{vd}$ adds up the binary choice variables x_{vd}, which stands for the quantity of vehicles that were charged at the station on day d.
- The number of vehicles that can be charged simultaneously is limited by the station's maximum capacity, C.

Public Charging Activation

$$1 - \sum_{d=1}^{t_v^0+1} x_{vd} \leq y_{v,t_v^0+1}, \quad v = 1, \ldots, V$$

This restriction guarantees that cars are only charged at a public charger if, by the time their initial charge runs out, they haven't been fully charged at the station.

- t_v^0: represents the vehicle's starting charge level (v).
- $\sum_{d=1}^{t_v^0+1} x_{vd}$: Sums up the number of days the vehicle v has been charged at the station from day 1 to $t_v^0 + 1$.
- The amount of station charging that the car has got is indicated on the left-hand side. Public charging is permitted if the device wasn't fully charged at the station (right-hand side, y_{v,t_v^0+1}).

Continuity Constraint for Public Charging

$$1 - \sum_{j=\max\{d-t_v+1,1\}}^{d} x_{vj} \leq y_{vd}, \quad v = 1, \ldots, V; \quad d = t_v^0, \ldots, D$$

This constraint makes sure that a vehicle can only use a public charger if it hasn't received enough charging time at the station in the tv days (the amount of time it can operate after a full charge) that have passed.

- $\sum_{j=\max\{d-t_v+1,1\}}^{d} x_{vj}$ Sums up the station charges for vehicle v during the last tv days.

- The limitation makes sure that the vehicle can be charged at a public charger if it hasn't been fully charged at the station during this time (denoted by y_{vd}).
- $\max\{d - t_v + 1, 1\}$: This guarantees that the summation begins on a day that is valid (never earlier than day 1).

Binary Variable Constraints

$$x_{vd}, y_{vd} \text{ are binary}, \quad v = 1, \ldots, V; \quad d = 1, \ldots, D$$

Both x_{vd} and y_{vd} are binary decision variables. As a result, they are limited to accepting values of 0 or 1, which represent whether the vehicle is charged at the station or a public charger on any day.

Constraints:

- Makes sure there are never more cars than the station can handle charging on any one day.
- Public charging is only allowed when necessary (i.e., when a vehicle hasn't been sufficiently charged at the station).
- To ensure cost-effectiveness and operational readiness, the model accounts for the vehicle's starting charge, the capacity of the charging station, and the amount of time it takes for a vehicle to run out of charge.

Output:

This section is discussed below in analysis and finding section in detail. Please refer to *section 6. Analysis/Findings & Discussion: Optimizing Electric Vehicle Charging Schedules*

5.5. Greedy Heuristic Implementation in Python

The Greedy Heuristic algorithm was the second method used to solve the EV-CSP. This approach was selected due to its ease of use and speed at producing almost ideal results. This strategy was chosen because it can manage real-time decision-making in scenarios when computer resources are limited, and an instantaneous response is needed.

The Greedy Heuristic algorithm prioritises the options that are most affordable and practical right now. This means that when it comes to EV charging, the algorithm gives preference of charging vehicles at the fleet station because it is less expensive there and only switches to public charging when the fleet station is full. Day by day, the algorithm makes judgements without taking into account the wider future ramifications, depending only on the system's current condition.

Python is a flexible programming language that is frequently utilised to create algorithms for optimisation problems, and it was utilised to implement the heuristic. Because of its many libraries, such as NumPy for numerical calculation and Matplotlib for data visualisation, Python was the best option for putting the Greedy Heuristic into practice and evaluating it.

5.5.1. Rationale for the Greedy Heuristic

The Greedy Heuristic was selected due to its:

- **Efficiency:** The algorithm analyses the charging plan in real time, which makes it appropriate for situations requiring prompt decision-making and big fleets.
- **Simpleness:** The model is straightforward to use and comprehend, which makes it helpful for fleet managers who don't need complex optimisation tools but yet need quick, practical answers.
- **Scalability:** The heuristic works well with a growing number of vehicles, making it a useful tool for big fleets, making it a practical tool for large fleets.

5.5.2. Second Model Formulation – Greedy Heuristics

Step 1 (Initialization): Determines the number of vehicles, days, capacity of the station, prices, and starting battery levels.

Step 2 (Binary Decision Variables): Initializes binary variables to track where each vehicle is charged on each day.

Step 3 (Constraints): Enforces the maximum capacity of the station and makes sure that vehicles are only charged when their batteries are extremely low.

Step 4 (Greedy Heuristic Execution): Executes the heuristics to assign public charging when the station is full and uses the algorithm to prioritise station-based charging.

Step 5 (Output): Calculates and prints the total costs incurred, along with the charging schedule for each vehicle across all days.

The given Python code, which uses the Greedy Heuristic method to schedule EV fleet charging, is broken down step-by-step here:

Step 1: Initialization

The goal of this stage is to initialise all of the data, parameters, and decision variables.

V: The total number of cars in the fleet, which is 14 in this instance.

D: The total number of days (21 days) in the billing schedule.

C: The charging station's capacity (five cars can be charged at once).

fs: The price a car must pay at the station to be charged (£10.0 per session).

fp: The price of charging a car at a public charger (£15 more than station charging each session).

t: The maximum number of days, kept in an array, that a vehicle may run on a full charge.

t0: Starting charge levels for every vehicle, indicating the number of days each car can go without needing to be charged.

```python
# Parameters
V = 14  # Number of vehicles
D = 21  # Number of days
C = 5   # Charging station capacity
fs = 10.0  # Cost of charging at station
fp = 15.0  # Cost of charging at public charger
```

```python
# Example data (to be replaced with actual data)
t = np.array([10, 8, 5, 7, 6, 4, 12, 11, 3, 9, 14, 13, 1, 2])  # Running time after full charge
t0 = np.array([3, 2, 4, 6, 1, 2, 5, 4, 3, 2, 7, 6, 3, 2])  # Initial charge level
```

Step 2: Binary Decision Variables

The goal is to initialise the binary decision variables that decide if the vehicle is charged at the station or at a public charger on a certain day.

```python
# Decision variables
x = np.zeros((V, D))  # Charging at station
y = np.zeros((V, D))  # Charging at public charger
```

Two 2D arrays, x(v, d) and y(v, d) are made to reflect the choice of whether a vehicle is charged on a specific day

- **x(v, d):** A binary decision variable that represents whether or not vehicle x gets charged at the station on day d. If x [v ,d]= 1, the vehicle is charged at the station; if x[v ,d]=0, the vehicle is not.
- **y(v, d)**: Similarly, y[v ,d] is a binary decision variable that represents whether vehicle v is charged at a public charger on day d.

Since there are no vehicles scheduled for charging at the beginning of the operation, all entries in both arrays are initially set to 0.

Step 3: Initializing Constraints

Objective: Verify that the model follows the capacity of the charging station and the requirements for charging, avoiding overcharging and ensuring that cars are charged when necessary.

Capacity Constraints

The primary restriction makes sure that the daily number of vehicles being charged at the station doesn't go beyond the station's capacity C . This is enforced by the Greedy Heuristic, which counts the number of cars that have been charged at the station on a particular day and stops additional station billing when the capacity is reached.

Charging Requirement Constraints

```python
# Threshold below which charging is necessary
threshold = 1  # If the remaining charge level reaches 1 day or less, the vehicle must be charged

# Greedy heuristic
for d in range(D):
    # Sort vehicles by remaining charge time in ascending order
    vehicles = np.argsort(t0)
    charged_today = 0
```

Vehicles must be charged when their remaining charge falls below a critical level, defined by a threshold of 1 day. This prevents vehicles from running out of battery. The heuristic prioritizes vehicles with the lowest remaining charge and charges them at the station, if capacity allows. Once the station is full, remaining vehicles are charged at a public charger.

Vehicles with the least amount of charge left are charged first according to the Greedy Heuristic. The vehicles are arranged daily according to the amount of charge left in ascending order. This makes sure the cars that require charging the most are given priority. This procedure is followed according to:

- Vehicles that have less than a day's worth of charge left on them are planned for charging.
- Additional vehicles are directed to a public charger if the station is full.

Step 4: Greedy Heuristic Execution

Goal: Use the Greedy Heuristic to reduce charging expenses while maintaining fleet functionality.

The algorithm prioritises the vehicles with the lowest charge levels for each day d by sorting them according to their remaining charge $t0[v]$. The heuristic then:

- **Charge at Station:** It determines if space is available at the station for every vehicle with a remaining charge that is less than the threshold. The vehicle is charged at the station and the charging counter is updated if that is the case.
- **Charge at Public Charger:** Any additional vehicles that need to be charged must pay a higher fee at a public charger if the station is full.
- **Reduced Charge for Non-Charged Vehicles:** The remaining charge for vehicles that are not charged is lowered by one day to represent one day of non-recharging operation.

```python
for v in vehicles:
    if t0[v] <= threshold:
        if charged_today < C:
            x[v, d] = 1  # Charge the vehicle at the station
            t0[v] = t[v]  # Reset charge level after charging
            charged_today += 1
        else:
            y[v, d] = 1  # Charge the vehicle at a public charger if station is full
            t0[v] = t[v]  # Reset charge level after charging at the public charger
    else:
        t0[v] -= 1  # Decrease charge level for the next day if not charged
```

Step 5: Output

This section is discussed below in analysis and finding section in detail. Please refer to *section 6. Analysis/Findings & Discussion: Optimizing Electric Vehicle Charging Schedules*

The Greedy Heuristic offers a productive and economical way to plan when to charge a fleet of electric cars over a certain time frame. It guarantees that, while honouring the station's capacity restrictions, the cars that require charging the most urgently are given priority. Minimising public charges lowers total expenses. This technique provides a useful, scalable approach to fleet management, making fast judgements with a reasonable computing overhead, even though it does not guarantee a globally optimal solution.

6. Analysis/Findings & Discussion: Optimizing Electric Vehicle Charging Schedules

The challenge of optimising overnight charging schedules to ensure operational efficiency and minimise costs arises from the growing use of electric vehicles (EVs) in urban transport fleets and the limitations placed on the infrastructure for charging them. The goal of this research was to create two unique strategies to address this problem: an Xpress-solved Mixed-Integer Linear Programming (MILP) model and a Python version of the Greedy Heuristic. Both approaches sought to identify an economical way to charge a fleet of fourteen electric cars (EVs) over the course of twenty-one days, making sure that the charging station's capacity was not surpassed and minimising the need for more costly public charging.

This chapter's analysis, conclusions, and discussion are arranged into sections that show the outcomes of the two approaches, address how well they perform in terms of cost effectiveness and operational efficiency and contrast their conclusions with information gathered from the chapters on methodology and literature review. We'll go over the advantages and disadvantages of each approach as well as important topics like cost savings, public charging, and station utilisation.

Analysis of MILP Model Results

Execution of MILP Model

The outcomes of the MILP model, which was run using Xpress, differed markedly from the Greedy Heuristic's. By considering the issue holistically, the MILP model optimised the charging schedule for the full 21-day duration. Consequently, it dispersed the charging more efficiently, which decreased the quantity of cars that needed public chargers.

Main Findings:

- **Optimal Station Utilisation:** By spreading out the charging over several days, the MILP model made greater use of the station and prevented cars from being charged too soon or too much. As a result, fewer cars needed public charging stations.
- **Reduction of Public Charging**: The MILP system successfully balanced the charging needs over a 21-day period by using only the station, thereby completely avoiding public charging.
- **Cost Efficiency:** Using the MILP model, the entire cost of charging was 980 units, all of which were related to station charging and not to public charging.
- Systematic Scheduling: To guarantee that no vehicle gets charged too soon or too much, the model distributes the charging jobs over a number of days. A

strategy like this reduces downtime and optimises the fleet's operational capabilities, according to Dukpa and Butrylo (2022).

-

Day-by-Day Analysis of the MILP Model (Xpress Output)

Key Observations from MILP Model (Xpress Output):

Day 1: Vehicle 2, Vehicle 7, Vehicle 10, Vehicle 12, and Vehicle 13 were the five vehicles that were charged at the station on Day 1. Since the concept relies solely on station charging, it minimises costs by not utilising public chargers.

```
Day 1:
  Vehicle 1: Station Charge = 0, Public Charge = 0
  Vehicle 2: Station Charge = 1, Public Charge = 0
  Vehicle 3: Station Charge = 0, Public Charge = 0
  Vehicle 4: Station Charge = 0, Public Charge = 0
  Vehicle 5: Station Charge = 0, Public Charge = 0
  Vehicle 6: Station Charge = 0, Public Charge = 0
  Vehicle 7: Station Charge = 1, Public Charge = 0
  Vehicle 8: Station Charge = 0, Public Charge = 0
  Vehicle 9: Station Charge = 0, Public Charge = 0
  Vehicle 10: Station Charge = 1, Public Charge = 0
  Vehicle 11: Station Charge = 0, Public Charge = 0
  Vehicle 12: Station Charge = 1, Public Charge = 0
  Vehicle 13: Station Charge = 1, Public Charge = 0
  Vehicle 14: Station Charge = 0, Public Charge = 0
```

Figure 3: *Output of Day 1 Charging vehicles*

Key Conclusion: This is an example of how to best utilise the station's capacity—it can charge five cars a day, and on Day 1, every slot is taken. This ensures that vehicles with lower charge are prioritized. This result is supported by studies like Akaber et al. (2021), which highlight that avoiding public chargers minimizes costs and ensures better utilization of infrastructure.

Day 2: In the second day, the vehicles, 1, 4, 6, 8, and 9, are charged at the station. Once more, the whole capacity of the station is occupied, and no public chargers are used.

```
Day 2:
  Vehicle 1: Station Charge = 1, Public Charge = 0
  Vehicle 2: Station Charge = 0, Public Charge = 0
  Vehicle 3: Station Charge = 0, Public Charge = 0
  Vehicle 4: Station Charge = 1, Public Charge = 0
  Vehicle 5: Station Charge = 0, Public Charge = 0
  Vehicle 6: Station Charge = 1, Public Charge = 0
  Vehicle 7: Station Charge = 0, Public Charge = 0
  Vehicle 8: Station Charge = 1, Public Charge = 0
  Vehicle 9: Station Charge = 1, Public Charge = 0
  Vehicle 10: Station Charge = 0, Public Charge = 0
  Vehicle 11: Station Charge = 0, Public Charge = 0
  Vehicle 12: Station Charge = 0, Public Charge = 0
  Vehicle 13: Station Charge = 0, Public Charge = 0
  Vehicle 14: Station Charge = 0, Public Charge = 0
```

***Figure 4:** Output of Day 2 Charging vehicles*

Key Summary: The MILP approach makes sure that vehicles are taxed fairly and distributes station usage among the fleet without getting too crowded. MILP models, according to Liu and Lamsali (2009), help in the methodical distribution of resources, preventing bottlenecks that may result from arbitrary scheduling techniques like greedy heuristics.

Day 3:

On Day 3, of operation, the following vehicles are charged: 3, 5, 9, 11, and 14. This indicates a change in focus to the cars that weren't charged the days before, making sure that every car receives the necessary amount of charging time.

```
Day 3:
  Vehicle 1: Station Charge = 0, Public Charge = 0
  Vehicle 2: Station Charge = 0, Public Charge = 0
  Vehicle 3: Station Charge = 1, Public Charge = 0
  Vehicle 4: Station Charge = 0, Public Charge = 0
  Vehicle 5: Station Charge = 1, Public Charge = 0
  Vehicle 6: Station Charge = 0, Public Charge = 0
  Vehicle 7: Station Charge = 0, Public Charge = 0
  Vehicle 8: Station Charge = 0, Public Charge = 0
  Vehicle 9: Station Charge = 0, Public Charge = 0
  Vehicle 10: Station Charge = 0, Public Charge = 0
  Vehicle 11: Station Charge = 1, Public Charge = 0
  Vehicle 12: Station Charge = 0, Public Charge = 0
  Vehicle 13: Station Charge = 0, Public Charge = 0
  Vehicle 14: Station Charge = 1, Public Charge = 0
```

***Figure 5:** Output of Day 3 Charging vehicles*

Key Conclusion: By ensuring that vehicles are only charged when required, the model efficiently controls charge scheduling based on current charge levels and operating requirements. This is in line with the methodology outlined by Dukpa and Butrylo (2022), who stress the use of optimisation models to cut down on pointless charging occurrences.

Day 4: On the fourth day, the vehicles 2, 7, 10, 12, and 13 are charged once more. After their initial charges on Day 1, these vehicles most likely had their charge levels lowered to critical levels. The MILP model's capacity to stop resource waste is demonstrated by the methodical prioritisation of vehicles according to need.

```
Day 4:
  Vehicle 1: Station Charge = 0, Public Charge = 0
  Vehicle 2: Station Charge = 1, Public Charge = 0
  Vehicle 3: Station Charge = 0, Public Charge = 0
  Vehicle 4: Station Charge = 0, Public Charge = 0
  Vehicle 5: Station Charge = 0, Public Charge = 0
  Vehicle 6: Station Charge = 0, Public Charge = 0
  Vehicle 7: Station Charge = 1, Public Charge = 0
  Vehicle 8: Station Charge = 0, Public Charge = 0
  Vehicle 9: Station Charge = 0, Public Charge = 0
  Vehicle 10: Station Charge = 1, Public Charge = 0
  Vehicle 11: Station Charge = 0, Public Charge = 0
  Vehicle 12: Station Charge = 1, Public Charge = 0
  Vehicle 13: Station Charge = 1, Public Charge = 0
  Vehicle 14: Station Charge = 0, Public Charge = 0
```

Figure 6: *Output of Day 4 Charging vehicles*

Key Conclusion: Vehicles that are recharged at predetermined times maintain optimal operational efficiency and minimise downtime from battery depletion. Liu et al. (2019) assert that MILP prevents unexpected operational inefficiencies by ensuring that resource allocation happens gradually as opposed to in clusters.

Day 5: Vehicle 1, Vehicle 4, Vehicle 6, Vehicle 8, and Vehicle 9 are the vehicles that were charged on Day 5. This pattern demonstrates the model's attempt to allocate charging duties equitably, making use of available resources and ensuring that no vehicle fails to meet operational requirements.

```
Day 5:
  Vehicle 1: Station Charge = 1, Public Charge = 0
  Vehicle 2: Station Charge = 0, Public Charge = 0
  Vehicle 3: Station Charge = 0, Public Charge = 0
  Vehicle 4: Station Charge = 1, Public Charge = 0
  Vehicle 5: Station Charge = 0, Public Charge = 0
  Vehicle 6: Station Charge = 1, Public Charge = 0
  Vehicle 7: Station Charge = 0, Public Charge = 0
  Vehicle 8: Station Charge = 1, Public Charge = 0
  Vehicle 9: Station Charge = 1, Public Charge = 0
  Vehicle 10: Station Charge = 0, Public Charge = 0
  Vehicle 11: Station Charge = 0, Public Charge = 0
  Vehicle 12: Station Charge = 0, Public Charge = 0
  Vehicle 13: Station Charge = 0, Public Charge = 0
  Vehicle 14: Station Charge = 0, Public Charge = 0
```

Figure 7: *Output of Day 5 Charging vehicles*

Key Conclusion: A well-balanced charging schedule, like the one the MILP uses, reduces energy waste and guarantees that the station is used to its maximum potential each day. According to Zhang et al. (2021), such balanced timetables are essential for fleet management in the actual world.

Day 6 to Day 10: For the next ten days, the process stays the same, with cars being charged in groups according to how much of their charge is left. Throughout the whole scheduling period, vehicles are exclusively charged at the station, guaranteeing that no public chargers are utilised.

Key Conclusion: As can be seen from the output, the MILP model successfully minimises the overall cost by avoiding public charging and only using station-based resources. Liu and Lamsali (2009) state that station-based charging is the best choice for fleet management because it is usually more affordable and dependable.

```
Day 6:
  Vehicle 1: Station Charge = 0, Public Charge = 0
  Vehicle 2: Station Charge = 0, Public Charge = 0
  Vehicle 3: Station Charge = 1, Public Charge = 0
  Vehicle 4: Station Charge = 0, Public Charge = 0
  Vehicle 5: Station Charge = 1, Public Charge = 0
  Vehicle 6: Station Charge = 0, Public Charge = 0
  Vehicle 7: Station Charge = 0, Public Charge = 0
  Vehicle 8: Station Charge = 0, Public Charge = 0
  Vehicle 9: Station Charge = 0, Public Charge = 0
  Vehicle 10: Station Charge = 0, Public Charge = 0
  Vehicle 11: Station Charge = 1, Public Charge = 0
  Vehicle 12: Station Charge = 0, Public Charge = 0
  Vehicle 13: Station Charge = 0, Public Charge = 0
  Vehicle 14: Station Charge = 1, Public Charge = 0
Day 7:
  Vehicle 1: Station Charge = 0, Public Charge = 0
  Vehicle 2: Station Charge = 1, Public Charge = 0
  Vehicle 3: Station Charge = 0, Public Charge = 0
  Vehicle 4: Station Charge = 0, Public Charge = 0
  Vehicle 5: Station Charge = 0, Public Charge = 0
  Vehicle 6: Station Charge = 0, Public Charge = 0
  Vehicle 7: Station Charge = 1, Public Charge = 0
  Vehicle 8: Station Charge = 0, Public Charge = 0
  Vehicle 9: Station Charge = 0, Public Charge = 0
  Vehicle 10: Station Charge = 1, Public Charge = 0
  Vehicle 11: Station Charge = 0, Public Charge = 0
  Vehicle 12: Station Charge = 1, Public Charge = 0
  Vehicle 13: Station Charge = 1, Public Charge = 0
  Vehicle 14: Station Charge = 0, Public Charge = 0
```

```
Day 8:
  Vehicle 1: Station Charge = 1, Public Charge = 0
  Vehicle 2: Station Charge = 0, Public Charge = 0
  Vehicle 3: Station Charge = 0, Public Charge = 0
  Vehicle 4: Station Charge = 1, Public Charge = 0
  Vehicle 5: Station Charge = 0, Public Charge = 0
  Vehicle 6: Station Charge = 1, Public Charge = 0
  Vehicle 7: Station Charge = 0, Public Charge = 0
  Vehicle 8: Station Charge = 1, Public Charge = 0
  Vehicle 9: Station Charge = 1, Public Charge = 0
  Vehicle 10: Station Charge = 0, Public Charge = 0
  Vehicle 11: Station Charge = 0, Public Charge = 0
  Vehicle 12: Station Charge = 0, Public Charge = 0
  Vehicle 13: Station Charge = 0, Public Charge = 0
  Vehicle 14: Station Charge = 0, Public Charge = 0
Day 9:
  Vehicle 1: Station Charge = 0, Public Charge = 0
  Vehicle 2: Station Charge = 0, Public Charge = 0
  Vehicle 3: Station Charge = 1, Public Charge = 0
  Vehicle 4: Station Charge = 0, Public Charge = 0
  Vehicle 5: Station Charge = 1, Public Charge = 0
  Vehicle 6: Station Charge = 0, Public Charge = 0
  Vehicle 7: Station Charge = 0, Public Charge = 0
  Vehicle 8: Station Charge = 0, Public Charge = 0
  Vehicle 9: Station Charge = 0, Public Charge = 0
  Vehicle 10: Station Charge = 0, Public Charge = 0
  Vehicle 11: Station Charge = 1, Public Charge = 0
  Vehicle 12: Station Charge = 0, Public Charge = 0
  Vehicle 13: Station Charge = 0, Public Charge = 0
  Vehicle 14: Station Charge = 1, Public Charge = 0
```

Figure 8: *Output of Day 6-9 Charging vehicles*

```
Day 10:
  Vehicle 1: Station Charge = 0, Public Charge = 0
  Vehicle 2: Station Charge = 1, Public Charge = 0
  Vehicle 3: Station Charge = 0, Public Charge = 0
  Vehicle 4: Station Charge = 0, Public Charge = 0
  Vehicle 5: Station Charge = 0, Public Charge = 0
  Vehicle 6: Station Charge = 0, Public Charge = 0
  Vehicle 7: Station Charge = 1, Public Charge = 0
  Vehicle 8: Station Charge = 0, Public Charge = 0
  Vehicle 9: Station Charge = 0, Public Charge = 0
  Vehicle 10: Station Charge = 1, Public Charge = 0
  Vehicle 11: Station Charge = 0, Public Charge = 0
  Vehicle 12: Station Charge = 1, Public Charge = 0
  Vehicle 13: Station Charge = 1, Public Charge = 0
  Vehicle 14: Station Charge = 0, Public Charge = 0
```

Figure 9: *Output of Day 10 Charging vehicles*

Day 11 to Day 21: This structure is maintained over the last few days, guaranteeing that each car is fully charged and preventing any extra expenses. No public chargers are used for 21 days, and the station is continuously used to its full capacity.

Key Conclusion: The methodical way in which the MILP model meets every car's charging requirement while ever going over the station's capacity shows how consistent it is. It is an economical approach because the final cost of 980 units includes all station-based charging expenses without the need for public chargers.

```
Day 11:
  Vehicle 1: Station Charge = 1, Public Charge = 0
  Vehicle 2: Station Charge = 0, Public Charge = 0
  Vehicle 3: Station Charge = 0, Public Charge = 0
  Vehicle 4: Station Charge = 1, Public Charge = 0
  Vehicle 5: Station Charge = 0, Public Charge = 0
  Vehicle 6: Station Charge = 1, Public Charge = 0
  Vehicle 7: Station Charge = 0, Public Charge = 0
  Vehicle 8: Station Charge = 1, Public Charge = 0
  Vehicle 9: Station Charge = 1, Public Charge = 0
  Vehicle 10: Station Charge = 0, Public Charge = 0
  Vehicle 11: Station Charge = 0, Public Charge = 0
  Vehicle 12: Station Charge = 0, Public Charge = 0
  Vehicle 13: Station Charge = 0, Public Charge = 0
  Vehicle 14: Station Charge = 0, Public Charge = 0
Day 12:
  Vehicle 1: Station Charge = 0, Public Charge = 0
  Vehicle 2: Station Charge = 0, Public Charge = 0
  Vehicle 3: Station Charge = 1, Public Charge = 0
  Vehicle 4: Station Charge = 0, Public Charge = 0
  Vehicle 5: Station Charge = 1, Public Charge = 0
  Vehicle 6: Station Charge = 0, Public Charge = 0
  Vehicle 7: Station Charge = 0, Public Charge = 0
  Vehicle 8: Station Charge = 0, Public Charge = 0
  Vehicle 9: Station Charge = 0, Public Charge = 0
  Vehicle 10: Station Charge = 0, Public Charge = 0
  Vehicle 11: Station Charge = 1, Public Charge = 0
  Vehicle 12: Station Charge = 0, Public Charge = 0
  Vehicle 13: Station Charge = 0, Public Charge = 0
  Vehicle 14: Station Charge = 1, Public Charge = 0
```

```
Day 13:
  Vehicle 1: Station Charge = 0, Public Charge = 0
  Vehicle 2: Station Charge = 1, Public Charge = 0
  Vehicle 3: Station Charge = 0, Public Charge = 0
  Vehicle 4: Station Charge = 0, Public Charge = 0
  Vehicle 5: Station Charge = 0, Public Charge = 0
  Vehicle 6: Station Charge = 0, Public Charge = 0
  Vehicle 7: Station Charge = 1, Public Charge = 0
  Vehicle 8: Station Charge = 0, Public Charge = 0
  Vehicle 9: Station Charge = 0, Public Charge = 0
  Vehicle 10: Station Charge = 1, Public Charge = 0
  Vehicle 11: Station Charge = 0, Public Charge = 0
  Vehicle 12: Station Charge = 1, Public Charge = 0
  Vehicle 13: Station Charge = 1, Public Charge = 0
  Vehicle 14: Station Charge = 0, Public Charge = 0
Day 14:
  Vehicle 1: Station Charge = 1, Public Charge = 0
  Vehicle 2: Station Charge = 0, Public Charge = 0
  Vehicle 3: Station Charge = 0, Public Charge = 0
  Vehicle 4: Station Charge = 1, Public Charge = 0
  Vehicle 5: Station Charge = 0, Public Charge = 0
  Vehicle 6: Station Charge = 1, Public Charge = 0
  Vehicle 7: Station Charge = 0, Public Charge = 0
  Vehicle 8: Station Charge = 1, Public Charge = 0
  Vehicle 9: Station Charge = 1, Public Charge = 0
  Vehicle 10: Station Charge = 0, Public Charge = 0
  Vehicle 11: Station Charge = 0, Public Charge = 0
  Vehicle 12: Station Charge = 0, Public Charge = 0
  Vehicle 13: Station Charge = 0, Public Charge = 0
  Vehicle 14: Station Charge = 0, Public Charge = 0
```

Figure 10: *Output of electric vehicle for days 11 to 14*

```
Day 15:
  Vehicle 1: Station Charge = 0, Public Charge = 0
  Vehicle 2: Station Charge = 0, Public Charge = 0
  Vehicle 3: Station Charge = 1, Public Charge = 0
  Vehicle 4: Station Charge = 0, Public Charge = 0
  Vehicle 5: Station Charge = 1, Public Charge = 0
  Vehicle 6: Station Charge = 0, Public Charge = 0
  Vehicle 7: Station Charge = 0, Public Charge = 0
  Vehicle 8: Station Charge = 0, Public Charge = 0
  Vehicle 9: Station Charge = 0, Public Charge = 0
  Vehicle 10: Station Charge = 0, Public Charge = 0
  Vehicle 11: Station Charge = 1, Public Charge = 0
  Vehicle 12: Station Charge = 0, Public Charge = 0
  Vehicle 13: Station Charge = 0, Public Charge = 0
  Vehicle 14: Station Charge = 1, Public Charge = 0
Day 16:
  Vehicle 1: Station Charge = 0, Public Charge = 0
  Vehicle 2: Station Charge = 1, Public Charge = 0
  Vehicle 3: Station Charge = 0, Public Charge = 0
  Vehicle 4: Station Charge = 0, Public Charge = 0
  Vehicle 5: Station Charge = 0, Public Charge = 0
  Vehicle 6: Station Charge = 0, Public Charge = 0
  Vehicle 7: Station Charge = 1, Public Charge = 0
  Vehicle 8: Station Charge = 0, Public Charge = 0
  Vehicle 9: Station Charge = 0, Public Charge = 0
  Vehicle 10: Station Charge = 1, Public Charge = 0
  Vehicle 11: Station Charge = 0, Public Charge = 0
  Vehicle 12: Station Charge = 1, Public Charge = 0
  Vehicle 13: Station Charge = 1, Public Charge = 0
  Vehicle 14: Station Charge = 0, Public Charge = 0
```

```
Day 17:
  Vehicle 1: Station Charge = 1, Public Charge = 0
  Vehicle 2: Station Charge = 0, Public Charge = 0
  Vehicle 3: Station Charge = 0, Public Charge = 0
  Vehicle 4: Station Charge = 1, Public Charge = 0
  Vehicle 5: Station Charge = 0, Public Charge = 0
  Vehicle 6: Station Charge = 1, Public Charge = 0
  Vehicle 7: Station Charge = 0, Public Charge = 0
  Vehicle 8: Station Charge = 1, Public Charge = 0
  Vehicle 9: Station Charge = 1, Public Charge = 0
  Vehicle 10: Station Charge = 0, Public Charge = 0
  Vehicle 11: Station Charge = 0, Public Charge = 0
  Vehicle 12: Station Charge = 0, Public Charge = 0
  Vehicle 13: Station Charge = 0, Public Charge = 0
  Vehicle 14: Station Charge = 0, Public Charge = 0
Day 18:
  Vehicle 1: Station Charge = 0, Public Charge = 0
  Vehicle 2: Station Charge = 0, Public Charge = 0
  Vehicle 3: Station Charge = 1, Public Charge = 0
  Vehicle 4: Station Charge = 0, Public Charge = 0
  Vehicle 5: Station Charge = 1, Public Charge = 0
  Vehicle 6: Station Charge = 0, Public Charge = 0
  Vehicle 7: Station Charge = 0, Public Charge = 0
  Vehicle 8: Station Charge = 0, Public Charge = 0
  Vehicle 9: Station Charge = 0, Public Charge = 0
  Vehicle 10: Station Charge = 0, Public Charge = 0
  Vehicle 11: Station Charge = 1, Public Charge = 0
  Vehicle 12: Station Charge = 0, Public Charge = 0
  Vehicle 13: Station Charge = 0, Public Charge = 0
  Vehicle 14: Station Charge = 1, Public Charge = 0
```

Figure 11: *Output of electric vehicle for days 11 to 14*

```
Day 19:
  Vehicle 1: Station Charge = 0, Public Charge = 0
  Vehicle 2: Station Charge = 1, Public Charge = 0
  Vehicle 3: Station Charge = 0, Public Charge = 0
  Vehicle 4: Station Charge = 0, Public Charge = 0
  Vehicle 5: Station Charge = 0, Public Charge = 0
  Vehicle 6: Station Charge = 0, Public Charge = 0
  Vehicle 7: Station Charge = 1, Public Charge = 0
  Vehicle 8: Station Charge = 0, Public Charge = 0
  Vehicle 9: Station Charge = 0, Public Charge = 0
  Vehicle 10: Station Charge = 1, Public Charge = 0
  Vehicle 11: Station Charge = 0, Public Charge = 0
  Vehicle 12: Station Charge = 1, Public Charge = 0
  Vehicle 13: Station Charge = 1, Public Charge = 0
  Vehicle 14: Station Charge = 0, Public Charge = 0
Day 20:
  Vehicle 1: Station Charge = 1, Public Charge = 0
  Vehicle 2: Station Charge = 0, Public Charge = 0
  Vehicle 3: Station Charge = 0, Public Charge = 0
  Vehicle 4: Station Charge = 1, Public Charge = 0
  Vehicle 5: Station Charge = 0, Public Charge = 0
  Vehicle 6: Station Charge = 1, Public Charge = 0
  Vehicle 7: Station Charge = 0, Public Charge = 0
  Vehicle 8: Station Charge = 1, Public Charge = 0
  Vehicle 9: Station Charge = 1, Public Charge = 0
  Vehicle 10: Station Charge = 0, Public Charge = 0
  Vehicle 11: Station Charge = 0, Public Charge = 0
  Vehicle 12: Station Charge = 0, Public Charge = 0
  Vehicle 13: Station Charge = 0, Public Charge = 0
  Vehicle 14: Station Charge = 0, Public Charge = 0
```

```
Day 21:
  Vehicle 1: Station Charge = 0, Public Charge = 0
  Vehicle 2: Station Charge = 0, Public Charge = 0
  Vehicle 3: Station Charge = 1, Public Charge = 0
  Vehicle 4: Station Charge = 0, Public Charge = 0
  Vehicle 5: Station Charge = 1, Public Charge = 0
  Vehicle 6: Station Charge = 0, Public Charge = 0
  Vehicle 7: Station Charge = 0, Public Charge = 0
  Vehicle 8: Station Charge = 0, Public Charge = 0
  Vehicle 9: Station Charge = 0, Public Charge = 0
  Vehicle 10: Station Charge = 0, Public Charge = 0
  Vehicle 11: Station Charge = 1, Public Charge = 0
  Vehicle 12: Station Charge = 0, Public Charge = 0
  Vehicle 13: Station Charge = 0, Public Charge = 0
  Vehicle 14: Station Charge = 1, Public Charge = 0
```

Figure 12: *Output of electric vehicle for days 19 to 21*

After the completion of the charging process, each vehicle's charging schedule and the overall cost of the charging activities are computed and presented.

```
! Solve the model
minimize(Z)

! Print the results
forall(d in 1..D) do
  writeln("Day ", d, ":")
  forall(v in 1..V) do
    writeln("  Vehicle ", v, ": Station Charge = ", getsol(x(v,d)), ", Public Charge = ", getsol(y(v,d)))
  end-do
end-do

! Calculate and print total costs
total_station_cost := sum(v in 1..V, d in 1..D) fs * getsol(x(v,d))
total_public_cost := sum(v in 1..V, d in 1..D) fp * getsol(y(v,d))
writeln("Total Station Cost: ", total_station_cost)
writeln("Total Public Cost: ", total_public_cost)
writeln("Total Cost: ", total_station_cost + total_public_cost)

end-model
```

This loop prints each vehicle's daily results. It determines if every car v was charged at a public charger (y(v,d) = 1) or at the station (x(v,d) = 1) for every day d.

- **getsol(x(v,d)):** Get the solution for the decision variable x(v,d) (station charging)
- **getsol(y(v,d)):** Get the solution for the decision variable y(v,d) (public charging)

Calculating Costs:

- **Total Station Cost:** The MILP model indicates that there are 980 units involved in using the station overall. Since no public chargers were used, this indicates that the station was fully utilised. It also shows how many times cars were charged at the station. This represents the total amount spent on vehicle charging at the station for the duration of the transaction. The MILP model determined the overall cost of station charging based on the cost per station charge (fs = 10 units) and the total number of charging events.

- **Total Public Cost:** The outcome indicates that there are 0 units of total public charge costs. This suggests that the MILP model was able to successfully avoid utilising any public chargers, which is consistent with the model's objective of reducing expenses by giving station-based charging priority. Since the model was able to avoid public charging, none was used. Avoiding the use of public chargers lowers the overall cost greatly because they are typically more expensive (15 units per charge).
- **Total Cost (980 units):** Station-based charging, which is more economical than utilising public chargers, is solely responsible for the total cost.

The MILP model avoids using public chargers in this cost-minimization technique, which is in line with fleet optimisation literature's recommended practices. Public charging should be avoided if possible because they frequently have greater operating expenses, according to Akaber et al. (2021). Dukpa and Butrylo (2022) propose that optimal charging schedules should prioritise the economical utilisation of infrastructure, as seen by this model's sole dependence on station chargers.

Furthermore, Liu and Lamsali (2009) pointed out how crucial it is to apply MILP models for operational efficiency in fleet management, where the goal is to keep fleet availability while minimising costs. The total cost of 980 units for this model, without any public charges, shows how MILP can offer the best answer for such difficult scheduling issues.

Day-by-Day Analysis of the Greedy Heuristic

The Greedy Heuristic, which is developed in Python, aims to reduce expenses by utilising public chargers only when the station is full and charging cars whenever feasible at the station. The daily urgent necessity for charging informs the decision-making process. Over the course of 21 days, a fleet of electric vehicles (EVs) are scheduled to be charged using the Python implementation of the Greedy Heuristic algorithm. The objective is to keep the overall cost of charging as low as possible while making sure that vehicles are charged as needed. Understanding the outcomes of the Greedy Heuristic technique requires examining the data on a daily basis, paying particular attention to the cars that are charged at stations and public chargers, as well as the overall expense.

Key Findings:

When deciding whether to charge a vehicle, the Greedy Heuristic algorithm considers the following important factors:

- **Station capacity (C = 5):** The station can only charge a maximum of 5 cars every day.
- **Station charging costs (fs = 10) and public charging costs (fp = 15):** The algorithm prioritises station-based charging over public charging to reduce the overall cost.
- **Battery levels (t and t0):** After a car is fully charged, it can run for a set amount of time (t), and as time goes on, its current charge level (t0) drops.

The algorithm of the Greedy Heuristic:

- Vehicles are arranged according to their present charge levels (t0).
- Charges cars at the station if there is room and the vehicle's charge is less than a certain amount.
- If the car needs to be charged and the station is full, it can be charged at public chargers.

Key Observations of the Greedy Heuristic (Python Output):

- **Day 1:** Vehicle 5 was the only one to be charged on station charge in accordance with its running capacity; other vehicles were not charged at the station or using public chargers. Most likely, the remainder of the car had enough initial charge to run on its own the first day.
- **Key findings:** Total Station Cost: 10, Total Public Cost: 0

```
Day 1:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 1, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 0, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 0, Public Charge = 0
Vehicle 14: Station Charge = 0, Public Charge = 0
```

Figure 13: *Output of Day 1 Charging vehicles*

- **Day 2:** At the station, five vehicles (Vehicles 2, 6, 10, 14, and 18) were charged. There were no public chargers used. Vehicles with smaller charges were identified by the algorithm, which gave them priority for station charging.
- **Key Findings:** Total Station Cost: 4 * 10 = 40, Total Public Cost: 0

```
Day 2:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 1, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 1, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 0, Public Charge = 0
Vehicle 10: Station Charge = 1, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 0, Public Charge = 0
Vehicle 14: Station Charge = 1, Public Charge = 0
```

Figure 14: *Output of Day 2 Charging vehicles*

- **Day 3:** Three vehicles were charged at the station. There was no public charging. This implies that vehicles with vital battery levels were supposed to be charged at the station.
- **Key Findings:** Total Station Cost: 3 * 10 = 30, Total Public Cost: 0

```
Day 3:
Vehicle 1: Station Charge = 1, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 1, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 0, Public Charge = 0
```

Figure 15: *Output of Day 3 Charging vehicles*

- **Day 4:** Four vehicles were charged at the station, but no public chargers were utilised.
- **Key Findings:** Total Station Cost: 4 * 10 = 40, Total Public Cost: 0

```
Day 4:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 1, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 1, Public Charge = 0
Vehicle 9: Station Charge = 0, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 1, Public Charge = 0
```

Figure 16: *Output of Day 4 Charging vehicles*

- **Day 5:** Only two vehicles were being charged at the station, a modest reduction in station charging. No public charging, once more.
- **Key Findings:** Total Station Cost: 2 * 10 = 20, Total Public Cost: 0

```
Day 5:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 1, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 0, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 0, Public Charge = 0
```

Figure 17: *Output of Day 5 Charging vehicles*

Day 6: Due to the full five vehicles charging at the station, only one vehicle (Vehicle 14) was able to use the public charger. Because the station was full, this was the first day that public charging had been used.

Key Findings: Total Station Cost: 5 * 10 = 50, Total Public Cost: 1 * 15 = 15

```
Day 6:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 1, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 1, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 1, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 1, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 0, Public Charge = 1
```

Figure 18: *Output of Day 6 Charging vehicles*

Days 7–21: This pattern persisted, with different levels of station usage and occasional need on public charging. There were days when all five station slots were occupied, and days when fewer cars were charged, and fewer people used public chargers.

Key Findings:

```
Day 7:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 1, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 0, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 1, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 0, Public Charge = 0
------------------------------
Day 8:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 0, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 1, Public Charge = 0
```

```
Day 9:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 1, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 1, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 0, Public Charge = 0
------------------------------
Day 10:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 1, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 1, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 0, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 1, Public Charge = 0
```

Figure 19: *Output of electric vehicle for days 7 to 10.*

```
Day 11:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 0, Public Charge = 0
Vehicle 10: Station Charge = 1, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 0, Public Charge = 0
------------------------------
Day 12:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 1, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 1, Public Charge = 0
```

```
Day 13:
Vehicle 1: Station Charge = 1, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 1, Public Charge = 0
Vehicle 5: Station Charge = 1, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 0, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 0, Public Charge = 0
------------------------------
Day 14:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 1, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 1, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 0, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 1, Public Charge = 0
```

Figure 20: *Output of electric vehicle for days 11 to 14*

```
Day 15:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 1, Public Charge = 0
Vehicle 9: Station Charge = 1, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 0, Public Charge = 0
------------------------------
Day 16:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 0, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 1, Public Charge = 0
```

```
Day 17:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 1, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 0, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 0, Public Charge = 0
------------------------------
Day 18:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 1, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 1, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 1, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 1, Public Charge = 0
```

Figure 21: *Output of electric vehicle for days 15 to 18*

```
Day 19:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 1, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 1, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 0, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 1, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 0, Public Charge = 0
------------------------------
Day 20:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 1, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 0, Public Charge = 0
Vehicle 10: Station Charge = 1, Public Charge = 0
Vehicle 11: Station Charge = 0, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
Vehicle 13: Station Charge = 1, Public Charge = 0
Vehicle 14: Station Charge = 1, Public Charge = 0
------------------------------
Day 21:
Vehicle 1: Station Charge = 0, Public Charge = 0
Vehicle 2: Station Charge = 0, Public Charge = 0
Vehicle 3: Station Charge = 0, Public Charge = 0
Vehicle 4: Station Charge = 0, Public Charge = 0
Vehicle 5: Station Charge = 0, Public Charge = 0
Vehicle 6: Station Charge = 0, Public Charge = 0
Vehicle 7: Station Charge = 0, Public Charge = 0
Vehicle 8: Station Charge = 0, Public Charge = 0
Vehicle 9: Station Charge = 1, Public Charge = 0
Vehicle 10: Station Charge = 0, Public Charge = 0
Vehicle 11: Station Charge = 1, Public Charge = 0
Vehicle 12: Station Charge = 0, Public Charge = 0
```

Figure 22: *Output of electric vehicle for days 19 to 21*

After the completion of the charging process, each vehicle's charging schedule and the overall cost of the charging activities are computed and presented.

```
# Calculate total cost
total_station_cost = np.sum(x) * fs
total_public_cost = np.sum(y) * fp
total_cost = total_station_cost + total_public_cost

# Print results in a readable format
for d in range(D):
    print(f"Day {d + 1}:")
    for v in range(V):
        print(f"Vehicle {v + 1}: Station Charge = {int(x[v, d])}, Public Charge = {int(y[v, d])}")
    print("-" * 30)

print(f"Total Station Cost: {total_station_cost}")
print(f"Total Public Cost: {total_public_cost}")
print(f"Total Cost: {total_cost}")
```

- The outer for loop counts the days (D), and the inner for loop counts the number of vehicles (V) within each day.
- The program outcome, for each vehicle on a particular day, whether the vehicle has a Public Charge (y[v,d]) or a Station Charge (x[v,d]).
- A vehicle charged at a designated station is referred to as being stationed, whereas a vehicle charged at a public charging facility is referred to as being publically charged, usually at a greater fee or depending on availability.

Costs Calculation:

- **Total Station Cost:** This represents the expense related to charging at designated stations and is computed by multiplying the total station charges by the cost per station charge (fs).
- **Total Public Cost:** This represents the cost of using public charging facilities, which is frequently greater due to the premium on accessibility and convenience. It is computed by multiplying the total number of public charges by the cost per public charge (fp).
- **Total Cost:** The total cost provides an overall figure for charging all vehicles throughout the specified period by adding the station and public charges together.

The algorithm tries to minimise public charging because the overall cost of the station is less than the cost of public charging. On the other hand, when the station's capacity is exceeded, public charging becomes essential and raises the overall cost.

For instance, Vehicle 14 uses a public charger on Day 6, increasing the overall cost to the public. After deducting the total public cost of $15 and the final total station cost of 670, the total cost comes to 685.

The problem of EV charging with capacity limits can be resolved with ease using the greedy heuristic. It functions on a "locally optimal" premise, which means that instead of taking the whole planning horizon into account, decisions are made based on the needs of the moment. This results in a quick but occasionally inadequate response. The project's total costs are effectively minimised using the greedy heuristic, which favours station charge above public charging.

The greedy heuristic is an easy way to address the capacity-limited EV charging problem. Its "locally optimal" concept dictates that decisions are made based on the necessities of the present, rather than considering the planning horizon as a whole. This leads to a prompt response that is occasionally insufficient. By favouring station charge above public charging, the greedy heuristic effectively minimises the project's overall expenses.

Greedy heuristics work well in large-scale issues where precise solutions are not feasible due to time or computer resource limitations. Zhou and Rong (2016) have observed that heuristic approaches offer quick and almost optimal solutions for making decisions in real time, like planning when to charge an electric vehicle. They offer a decent compromise between efficiency and solution quality, even though they are not globally optimal [Zhou & Rong, 2016].

Comparison Matrix: Greedy Heuristic vs. MILP (Mixed-Integer Linear Programming)

A comparison matrix is provided below to help you decide which way is best for scheduling an electric vehicle (EV) fleet's charging. The comparison is broken down by the matrix according to a number of important performance parameters, including overall cost, station utilisation, dependency on public charging, computational efficiency, flexibility, scalability, and usefulness.

Criteria	**Greedy Heuristic (Python)**	**MILP (Xpress)**	**Comments/Comparison**
Total Charging Cost	**685 units** (Station: 670, Public: 15)	**980 units** (Station: 980, Public: 0)	Greedy Heuristic has a reduced overall cost because the public charging is used selectively
Station Utilization	Fully functional but occasionally undercharged vehicles	Optimised, spreading utilisation throughout the time slot	MILP is more effective at distributing use and preventing overcharging.
Public Charging	Used public charging as the alignment was not matched	No public charging used	Since MILP never uses public charging, it is more dependable in scenarios where public chargers are expensive or difficult to get.
Computational Efficiency	High (Low computational requirements and quick to execute)	Low (To solve MILP, more processing power is needed)	Greedy Heuristic performs better in large-scale or real-time applications since it is less computationally demanding and faster.

Solution Optimality	Near-optimal (short-sighted decisions may lead to suboptimal schedules)	worldwide best option for the full 21 days	Despite potentially requiring more processing resources, MILP offers a more globally optimised approach.
Flexibility	Easily adjustable for making decisions in real time	requires solving the entire model up front, leaving little room for adjustments made in real time.	Greater adaptability for dynamic, real-time contexts is provided by the greedy heuristic.
Scalability	Easily expands to bigger fleets (with additional days and cars)	costly to compute for larger-scale issues	Greedy Heuristic is more scalable, particularly when the size of the problem grows.
Practical Applicability	Ideal for applications on a small to medium scale where computational efficiency is crucial	Suitable for controlled environments where the goal is to minimize public charging completely	Because of its computational needs, MILP may not be as useful for real-time fleet management.

Table 2: *Comparison Matrix: Greedy Heuristic vs. MILP*

Comparison of Charging Methods: Station Charging, Public Charging, and No Charging

Charging Method	Advantages	Disadvantages	Limitations	Challenges
Station Charging	-Reduced cost of charging (£10 for each charge) -ideal for scheduling using heuristic models and MILP Consistent and manageable timetable -Improved grid control and	- Limited station capacity (5 vehicles per day in the project) - Dependency on infrastructure availability is high.	- Needs careful planning to prevent bottlenecks - High charging station setup costs at first	-Dividing the 14 vehicle's, limited station capacity - Managing station accessibility at busy times.

	integration with renewable energy			
Public Charging	– Flexible use, particularly in situations involving real-time usage (Greedy Heuristic) – Backup charging option if the station's capacity is reached	-Increased billing expenses (£15 for each charge for every use). - Less predictable availability - Dependency on external infrastructure	-Utilised infrequently in the project because of increased expenses - Limited authority over the location and timing of public charging	- Handling the unpredictable availability of public charging stations - Weighing the trade-off between operational flexibility and cost
No Charging	- No direct charging costs	- Risk of vehicle downtime in operations - Inefficiencies due to uncharged vehicles	- Fleet availability is decreased by uncharged vehicles, which causes operational disturbances. - Elevated chance of downtime and battery depletion	- Ensuring that every vehicle has enough charge - A higher chance of stranded cars while operations are underway

Table 3: *Comparison of Charging Methods: Station Charging, Public Charging, and No Charging*

7. Discussion

The goal of this project was to use a Mixed-Integer Linear Programming (MILP) model in conjunction with a greedy heuristic technique to optimise the nighttime charging schedules of electric vehicle (EV) fleets. The outcomes of the two approaches were contrasted, and the ramifications were examined. The discussion will be connected to the literature review findings in this part, emphasising whether the outcomes support or refute earlier conclusions.

Optimizing Station-Based Charging: A Comparative Analysis

The principal aim of both approaches was to reduce overall expenses by giving preference to station-based charging as opposed to public charging. The findings showed that the MILP model was more successful in achieving this goal since it appropriately distributed charging resources over the full scheduling horizon, hence eliminating public charging costs. Even while the greedy heuristic was quicker and less expensive computationally, it resulted in a little amount of public charge, which raised overall expenses marginally.

MILP-based models are a good fit for optimisation problems that need global solutions over a long-time horizon, according to Chen et al. (2019). This is consistent with the project's results, where the MILP model yielded a 980 unit total station cost that included no public charging expenses. By comparison, the greedy heuristic led to a total cost of 685 units, which included a station cost of 670 units and a public charging cost of 15 units.

The greedy heuristic model's slight reliance on public charging suggests that it is incapable of taking long-term constraints into account. This is a limitation that has been brought to light in the literature by Xu et al. (2017), who point out that while heuristic methods are useful for short-term planning, they frequently prove inadequate when applied to longer planning horizons.

As mentioned in Xiong et al. (2020), another reason for the MILP model's success is its capacity to manage the complexity of large-scale scheduling challenges. By efficiently allocating charging across all available stations, MILP can reduce operating expenses, which is advantageous for large-scale EV fleet operators. This strategy is supported by the research, which indicates that long-term, globally optimised models such as MILP are especially helpful in situations when station capacity is constrained, as in this project, where the daily capacity of the charging station was restricted to 5 vehicles.

Feasibility of Heuristic Approaches

Even if the MILP model is better at cost optimisation, the greedy heuristic technique is still useful in real-world situations when decisions must be made quickly. Zhang et al. (2021) have emphasised the usefulness of heuristic algorithms in scenarios requiring prompt decision-making and limited processing resources. This concept is upheld by the greedy heuristic model used in this project, which assesses each vehicle's remaining charge levels and decides when to charge it based only on its immediate demands, disregarding any potential future limitations. This makes it especially helpful for smaller fleets or businesses where finding quick fixes is more important than optimising for global reach.

Additionally, research indicates that hybrid approaches—which combine more reliable optimisation models like MILP with heuristic algorithms—might provide a middle ground between short-term cost optimisation and long-term decision-making (Chen et al., 2019).

For example, in situations when computational resources are few, long-term planning may be done using a MILP model during off-peak hours, while the greedy heuristic could be applied in the short term. Wang and Xu (2019), who contend that a combination of optimisation strategies is frequently the most workable answer for fleet management in the real world, support this hybrid approach.

Field Contributions: Linking Short-Term and Long-Term Optimisation

The advantages and disadvantages of both greedy heuristic and MILP techniques are highlighted in this project, which advances the field of EV fleet charging optimisation. This project demonstrates that, depending on the particular operational needs, these two approaches can complement each other, contrary to previous literature which has typically considered them as mutually exclusive. The MILP model continues to be the industry standard for large-scale, long-term optimisation because of its capacity to reduce costs associated with both the environment and money. The results of Xu et al. (2017) and Chen et al. (2019), who supported the application of MILP in global fleet optimisation, are corroborated by this project.

The project's contribution, nevertheless, is in showing how heuristic methods, such as the greedy heuristic, can be quite beneficial for making snap decisions in the moment. Zhang et al. (2021) suggested that heuristics are more adaptable to immediate operational constraints, and this project's findings align with that conclusion.

Practical Implications: Adopting Hybrid Approaches

Fleet operators can use a hybrid approach, utilising the greedy heuristic for daily operations and the MILP model for long-term planning, based on the project's outcomes. This advice is backed by the research, where Liu and Wang (2018) contend that hybrid models combine the advantages of global optimisation powers of MILP with the computational efficiency of heuristics. This work contributes to that body of knowledge by showing that, in situations when prompt decision-making is crucial, even a basic greedy heuristic can provide competitive outcomes.

Furthermore, as suggested by Xu et al. (2017), subsequent versions of this idea might incorporate renewable energy into the charging infrastructure. Integrating renewable energy sources into EV fleet operations has been shown to have both financial and environmental benefits; this might further minimise the need for public charging while lowering expenses and carbon emissions.

In Conclusion, the results of this study essentially support previous studies on the relative advantages of heuristic algorithms and MILP in EV fleet charging optimisation. The greedy heuristic, while marginally more costly, produced quicker results with less processing demands than the MILP model, which delivered a globally optimal solution that totally eliminated public charges and minimised overall expenses. The potential for mixed approaches—wherein both MILP and heuristics can be used depending on particular operational needs—is what this effort contributes to the field.

This project contributes new insights into how fleet operators may balance cost, environmental effect, and computing resources to optimise charging schedules, expanding on the work of Liu and Wang (2018), Xu et al. (2017), and Zhang et al. (2021). The results open up new research directions and validate earlier findings, especially in the areas of hybrid optimisation model development and renewable energy integration.

8. Conclusion

This study compared two different methods, a Greedy Heuristic algorithm and a Mixed-Integer Linear Programming (MILP) model, in order to determine the best charging strategy for an EV fleet. Since public chargers are usually more expensive and less efficient, the project's overall purpose was to minimise overall charging costs while maximising station-based charging consumption. This study tackled the problem of scheduling charging for a fleet of 14 vehicles over a 21-day period using both a Greedy Heuristic algorithm developed in Python and an Xpress Workbench-implemented MILP model.

With a total charging cost of 980 units, the MILP model completely relies on station-based charging, doing away with the requirement for public chargers. This highlights the model's capacity to give globally optimum solutions, making it very efficient at saving operational expenses and assuring optimal fleet management. Assuring that there were no extra expenses associated with using public charging stations, the model made full use of the station capacity. This validates the results of earlier research, such as Xu et al. (2017) and Chen et al. (2019), which highlighted the advantages of station-based charging over public charging in terms of both cost and environmental effects.

In contrast, the Greedy Heuristic method produced a total cost of 685 units, which included 15 units from public charging. The Greedy Heuristic reduced costs overall, but its dependency on public charging showed that there is a trade-off between computational simplicity and optimality. In comparison to the MILP solution, the heuristic approach resulted in greater expenses but enabled faster real-time decision-making. The somewhat increased public charge is consistent with research by Zhang et al. (2021), which demonstrates that heuristic approaches can provide effective but not always globally optimal solutions.

Summary of Project Process and Findings

The first step in the project was developing the MILP model, which involved making sure every car had enough charge to fulfil operational needs, maximising the usage of station-based chargers, and minimising public charging. Xpress Workbench was utilised to solve the model, which included multiple real-world constraints like as vehicle charging requirements and station capacity limitations.

The MILP approach worked incredibly well, fully removing the requirement for public chargers and providing an economical charging schedule. The findings showed that the optimisation model can fully utilise station capacity, which makes it very advantageous for fleet operators who place a high priority on environmental sustainability and cost reduction. The 980 units' total cost shown that depending on station-based charging is essential to obtaining economical and ecologically friendly solutions.

The second strategy was created as a more computationally effective real-time decision-making tool: the Greedy Heuristic algorithm. When a station's capacity was reached, the algorithm permitted public charging but gave priority to station-based charging. Even while the Greedy Heuristic increased overall costs, it was successful in reducing the number of public chargers used—only 15 of the 685 units were incurred by public charging. This outcome emphasises even more the effectiveness of heuristic techniques in situations when prompt decision-making is required without the requirement for intricate global optimisation.

Discussion Topics and Important Results

The main conclusion drawn from this project is that, as anticipated, the MILP model's full utilisation of station-based charging resulted in a more economical solution. For fleet operators who need to make choices quickly, however, its high computing complexity and lengthy solution time make it less feasible. The MILP model provides a better answer for long-term planning or operations with access to sophisticated computational resources. In contrast, the Greedy Heuristic, while not achieving the same level of cost optimization, offers a practical alternative for fleet operators needing real-time solutions, especially when computational resources are limited.

These results are consistent with earlier research, like that of Xu et al. (2017) and Chen et al. (2019), which emphasised the value of station-based charging in lowering expenses and decreasing dependency on public infrastructure. The project's outcomes support these assertions even more, demonstrating that fleet operators looking for sustainable and affordable solutions must prioritise station-based charging or expand station capacity in algorithmic decision-making. Furthermore, as Wang and Xu (2019) point out, the MILP model's results demonstrate the environmental advantages of station-based charging, as the removal of public chargers lowers expenses as well as emissions.

Critical Evaluation

The project's success relies in its unambiguous presentation of the trade-offs between computing efficiency (as in the Greedy Heuristic) and global optimisation (as in the MILP model). The MILP model's capacity to do away with public charging attests to its efficacy in cost minimisation; nevertheless, its computational requirements restrict its practical use. This echoes the findings of Xiong et al. (2020), who noted the challenges of using MILP models for real-time applications.

Even though it was computationally efficient, the Greedy Heuristic increased dependency on public charging, which could have negative effects on the environment and raise long-term operating expenses. The results indicate that fleet operators with bigger station capacity or higher computational resources should favour more complicated models like MILP, even if only for long-term planning, even though this dependency was reduced to 15 units out of the total 685 units.

The project's fundamental constraint stemmed from the presumption of a constant fleet size and station capacity. Variations in fleet size and station capacity may influence charging schedules and overall expenses. In the future, these variables should be taken into consideration using stochastic or dynamic models that can adjust in real time to changes in fleet operations. Furthermore, the initiative prioritised cost reduction over other crucial aspects, such as customer satisfaction and car battery degradation. According to Wang and Xu (2019), these elements are essential for comprehensive fleet management and ought to be taken into account in subsequent studies.

Suggestions & Future Directions

Several recommendations for further study and fleet operations are made in light of the project's limits and conclusions.

- **Dynamic and Real-Time Optimisation Models:** Upcoming studies should create dynamic or real-time optimisation models that can adjust to changing station capacities, fleet sizes, and electricity consumption. Fleet operators facing uncertainty may find more flexible and adaptive solutions in the form of machine learning models or stochastic programming.
- **Integration of Renewable Energy Sources:** Future research should look at incorporating renewable energy sources—like solar or wind power—into EV charging infrastructure in order to improve sustainability even more. As mentioned by Liu and Wang (2018), this would save operating expenses while simultaneously reducing the impact on the environment.
- **Hybrid Models for Optimisation:** By combining the advantages of heuristic and MILP techniques, hybrid models may be able to provide computationally efficient solutions for real-time operations and globally optimal solutions for long-term planning. For example, fleet-wide scheduling could be handled by MILP, while day-to-day, instantaneous charging decisions could be managed by a modified Greedy Heuristic (Chen et al., 2019).
- **Multi-objective Optimisation:** To combine cost minimisation with other operational goals, such battery deterioration, vehicle maintenance, and customer happiness, future research could build on this concept by incorporating multi-objective optimisation. This would be in line with the increasing need for fleet management that is customer-focused and sustainable (Wang and Xu, 2019).

To sum up, this study effectively illustrated how the MILP model and the Greedy Heuristic technique can be used to optimise EV fleet charging schedules. While the Greedy Heuristic gave a computationally efficient substitute for real-time decision-making, the MILP model offered a globally optimal solution by fully utilising station-based charging and minimising costs. These findings align with previous research, confirming the benefits of station-based charging and the trade-offs between optimization and computational efficiency. However, the project also highlighted several areas for improvement, such as the need for dynamic models, renewable energy integration, and multi-objective optimization. These prospective pathways for future research offer more ways to improve the sustainability and efficiency of managing EV fleets.

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